

## MIDTERM EXAMINATION

This examination is take-home, open book, open notes. Feel free to consult any published reference you wish. Cite references if you need to use their results in answering the examination.

It is not useful to quote or paraphrase at length from Starr's *General Equilibrium Theory: An introduction*. In answering exam questions, state clearly any additional assumptions you need.

**Please do not consult or discuss with classmates, other students, faculty (except for Aislinn or Ross), or friends. OTHER PEOPLE ARE CLOSED.**

Answer any four (4) questions. All count equally.

ENJOY!!! GOOD LUCK!!

1. Using the notation of chapter 12 of Starr's *General Equilibrium Theory*:

*An introduction*, the production sector of the economy is described this way:

There is a finite family  $F$  of firms  $j \in F$ , each with a technology  $Y^j \subset \mathbb{R}^N$ .  
An allocation  $x^i, i \in H$ , is attainable if there is  $y^j \in Y^j, j \in F$ , so that  
 $0 \leq \sum_{i \in H} x^i \leq \sum_{j \in F} y^j + \sum_{i \in H} r^i$  (the inequalities hold co-ordinatewise). A production  
plan  $y^j, j \in F$ , is said to be attainable if  $0 \leq \sum_{j \in F} y^j + \sum_{i \in H} r^i$ .

New definition: An attainable production plan  $y^j$  is said to be technically  
efficient if there is no alternative attainable production plan  $w^j \in Y^j, j \in F$ , so that  
 $\sum_{j \in F} w^j \geq \sum_{j \in F} y^j$  with the strict inequality holding in some co-ordinate.

Assume:

Strong Monotonicity (every good is desirable): For each household  $i$   
 $\in H$ , with utility function  $u^i(\cdot)$ ,  $w, x \in X^i, w \geq x$  (co-ordinatewise),  
 $w \neq x$ , implies  $u^i(w) > u^i(x)$ .

Note (without proof) that strong monotonicity implies that general equilibrium  
prices are strictly positive. You may use this observation in the questions below.

Please establish two results:

(i) Under strong monotonicity, technical efficiency of an allocation is  
a necessary condition for Pareto efficiency.

(ii) Under strong monotonicity, let  $p, y^j, x^i, j \in F, i \in H$ , be a  
competitive equilibrium. Then  $y^j$  is technically efficient (strict positivity of prices  
may be helpful).

2. The following question assumes the pure exchange economy model and the growth of the economy by replication presented in Starr's *General Equilibrium Theory: An introduction*, Chapters 13 and 14. We use assumptions C.IV, C.V, C.VII, C.IX, throughout this question.

Define a new concept, the *balanced core*, as the set of allocations unblocked by *balanced* coalitions.

A blocking coalition  $S$  in economy  $Q-H$  will be said to *balanced*, if it fulfills two properties:

- it contains the same number of individuals of each type represented in the coalition. For example, a coalition of five households each of types 1, 3, and 7, and zero of all other types fulfills this condition; a coalition of five of type 1, four of type 3, one of type 7, and zero of all other types is not balanced.
- equal treatment within the coalition. For any blocking allocation, all households of the same type within the blocking balanced coalition receive the same allocation.

Note that every coalition in  $1-H$  (the original unreplicated economy), is balanced.

We wish to demonstrate that, unlike the core, the balanced core does not converge to the set of competitive equilibria.

Do parts (i), (ii), and (iii):

(i) The balanced core retains two properties of the core: inclusion of the competitive equilibrium (Theorem 13.1), and the equal treatment property (Theorem 14.1). Explain why (this should take no more than a few sentences).

(Question 2 continues next page)

(ii) Prove and explain

**Proposition:** Let  $\{x^{oh} \mid h \in H\}$  be in the core for  $Q = 1$  (the original unreplicated economy). Then  $\{x^{oh} \mid h \in H\}$  is in the balanced core of  $Q$ - $H$  for all positive integer values of  $Q$ .

Hint: Show that if  $x^{oh}$  is blocked by a balanced coalition in  $Q$ - $H$ , then it is blocked in  $1$ - $H$  (the original unreplicated economy).

(iii) The Proposition in part (ii) means that the balanced core does not converge to the (set of) competitive equilibrium(a). Explain this interpretation.

3. We developed the notion of an Edgeworth Box for the allocation of consumption between two households in Chapter 1 of Starr's *General Equilibrium Theory: An introduction*. We can use the same approach to describe the allocation of inputs to production. Factors of production are analagous to consumption goods in the (consumption) Edgeworth Box, output levels are analagous to household utilities, isoquants are analagous to indifference curves.

Let there be two inputs to production, X and Y, endowed in the amounts  $\bar{X}, \bar{Y} > 0$ . They are to be allocated between the production of outputs 1 and 2, in the amounts  $X^1, X^2, Y^1, Y^2$  subject to the constraints  $X^1 + X^2 = \bar{X}, Y^1 + Y^2 = \bar{Y}$ . They produce outputs 1 and 2 according to the production functions

$$Q^1 = F(X^1, Y^1) = [X^1 Y^1]^{1/2}, \quad Q^2 = G(X^2, Y^2) = [X^2 Y^2]^{1/3}.$$

(The superscripts  $1/2$  and  $1/3$  are exponents; the superscripts 1 and 2 indicate the product). The allocation of inputs to production is *technically efficient* if there is no reallocation of inputs that would allow an increase in output of 1 or 2 without a reduction in output of the other.

(a) Consider an Edgeworth Box with sides of length  $\bar{X}$  and  $\bar{Y}$ . Let opposite corners of the box depict two allocations, one with all resources going to produce 1 and the other with all resources going to produce 2. Describe the locus of tangencies of the isoquants. This locus represents the technically efficient allocation of resources to production. Explain why.

(b) Let the factors sell for  $p_x$  and  $p_y$ , with each firm choosing its input mix to minimize the cost of inputs for each level of output. Show that a factor market equilibrium will lie on the locus of tangencies.

(c) The production possibility set (bounded by the production frontier) in output (good 1 - good 2) space is defined as

$$PPS = \{(Q^1, Q^2) \mid Q^1 \leq F(X^1, Y^1) = [X^1 Y^1]^{1/2}, Q^2 \leq G(X^2, Y^2) = [X^2 Y^2]^{1/3}; X^1 + X^2 = \bar{X}, Y^1 + Y^2 = \bar{Y}\}$$

Describe this set. What is the relationship of the production frontier to the locus of isoquant tangencies in the Edgeworth Box?

4. The conventional economic model of the firm in intermediate economics textbooks has a U-shaped cost curve reflecting a small nonconvexity in the technology at low levels of output. This is contrary to the conventional general equilibrium model that typically requires convexity of production technology. Applying the general equilibrium model (e.g. of Starr's *General Equilibrium Theory*, chaps 4-12) to an economy with firms having U-shaped cost curves, can you conclude:

- a. there is no general equilibrium?
- b. even if a general equilibrium exists, the allocation of resources in equilibrium may not be Pareto efficient?
- c. there may not exist a general equilibrium, but it is possible that one exists?
- d. if a general equilibrium exists, then the cost curves were not really U-shaped; there is no nonconvexity in the technology?

Explain your answer in each part.

5. In discussing the relationship of saving to consumption in a monetary economy, Keynes writes

"An act of individual saving means --- so to speak --- a decision not to have dinner to-day. But it does *not* necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus it depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption...If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect might indeed be different."

--- J. M. Keynes, *The General Theory...*, chap. 16.

Keynes is arguing that a saving decision implies an intention to postpone consumption but that intention does not show up as a demand for future consumption in the economy. Can the difficulty Keynes notes ("depresses the business of preparing to-day's consumption without stimulating ... some future act of consumption") occur in an Arrow-Debreu economy with a full set of futures markets? Explain.